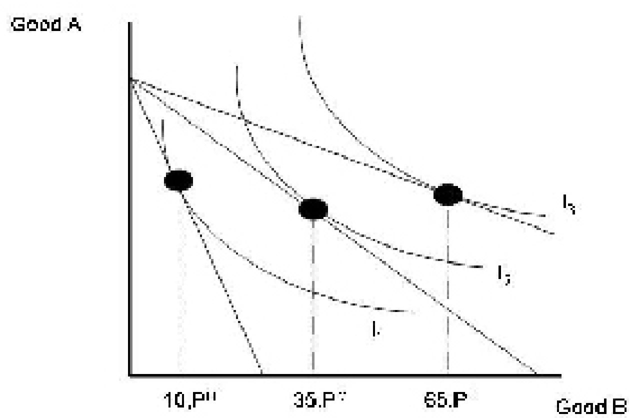
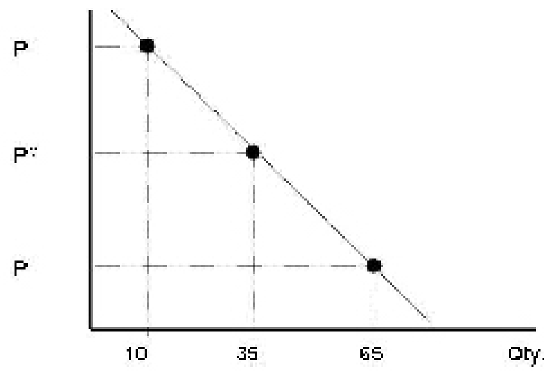
# 1 Deriving Demand Curves

We should be able to connect an individual's optimal bundle purchase to his demand curve. Recall that when creating a demand curve we hold everything in the world fixed except the price and quantity demanded of the good. In our two-good "world" with a fixed budget, "everything else" consists of the price of the other good and the individual's income. So we need to hold those two factors constant when deriving an individual's demand curve for a product.

Look at the picture below. There are three budget constraints, all starting from the same point on the good A axis. These 3 budget constraints correspond to a high price for good B $\left(P^H\right)$, a medium price for good B $\left(P^M\right)$, and a low price for good B $\left(P^L\right)$.



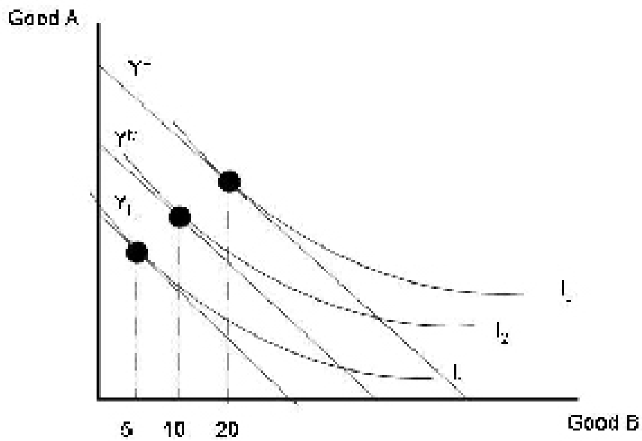

Notice that when the price of good B is $P^H$ that the consumer only purchases 10 units of good B at his optimal bundle. When the price of good B is $P^M$, the consumer now purchases 35 units of good B, and when the price of good B is $P^L$ the consumer purchases 65 units of good B. Notice that this is exactly what we would need to construct a demand curve for good B – the price and quantity pairs for the good. Plotting these points on a price-quantity axis gives us:



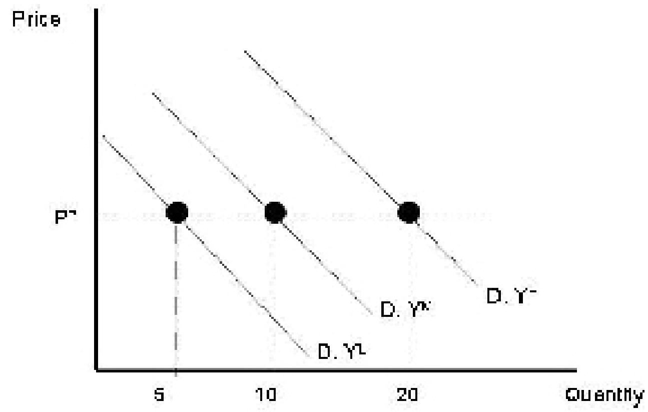

Notice that we get a nice, intuitive downward-sloping demand curve.

## 1.1 Income changes and demand curves

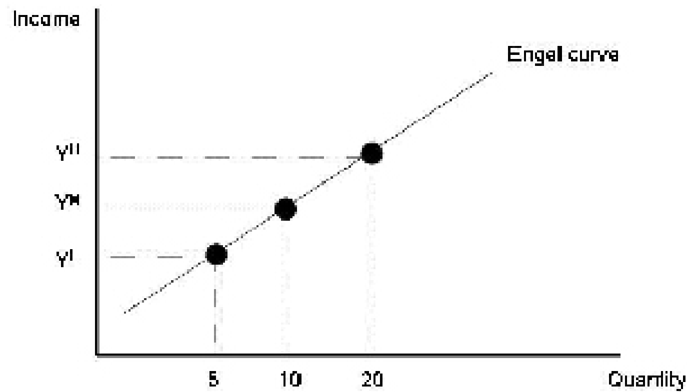
We know from principles of micro that an increase in income (at least for normal goods) will cause the demand curve to increase (shift to the right). We can also use indifference curve analysis to show this fact. The picture below shows three budget constraints, each with a different income level $(Y_L, Y_M, Y_H)$. We then find the consumers optimal consumption bundle for each income level. Notice that we will be keeping the prices of goods A and B the same.




Now, suppose that we were to plot the points on a price-quantity graph for Good B. The key is to realize that the price of Good B has NOT changed. Suppose the price is some price $P^*$. Then we would get the following graph, with three distinct demand curves corresponding to $Y^L$, $Y^M$, and $Y^H$. Note that we only have one point on each of the demand curves – if we wanted to get more points we would need to look at a fixed income level (either $Y^L$ or $Y^M$ or $Y^H$) and change the price of Good B.

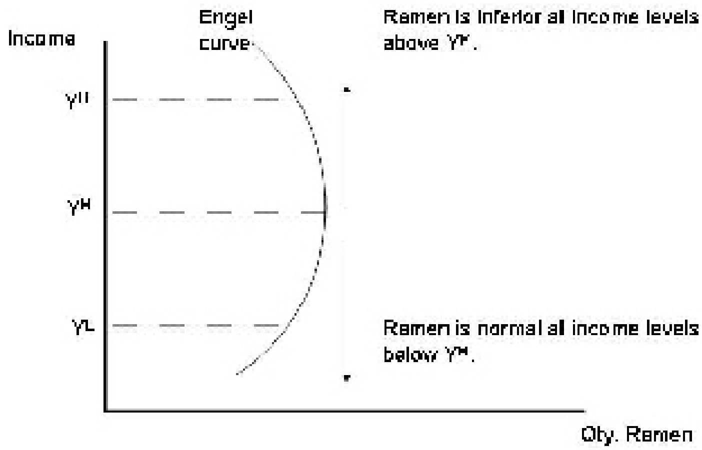



Since the demand for Good B increases when we increase income, we have a normal good. We can also show the relationship between income and quantity demanded on a separate graph. Define an Engel curve as the graphical representation of the relationship between a consumer's income and his quantity demanded. We will place income on the y-axis and quantity demanded on the x-axis. The Engel curve for Good B is shown below. Note that if the Engel curve is upward-sloping then the good is a normal good; if the Engel curve is downward-sloping the good is an inferior good.




### 1.1.1 Goods that are normal and inferior

It is highly possible that some goods may be both normal goods and inferior goods, depending on the range of income. To someone who has very little income, Ramen noodles may be a normal good if that person is given more income – they can now consume one additional meal. But if that same consumer receives a large enough increase in income, then he may consume less Ramen noodles and switch to consuming higher quality foods. In a case like this, the Engel curve for the good will be backward-bending. Over the part of the curve with the increasing slope the good is a normal good, but when income becomes high enough (which is at the income level $Y^M$ in the picture), the consumer starts to shift away from purchasing the good when he receives more income, which means that the good is now inferior.

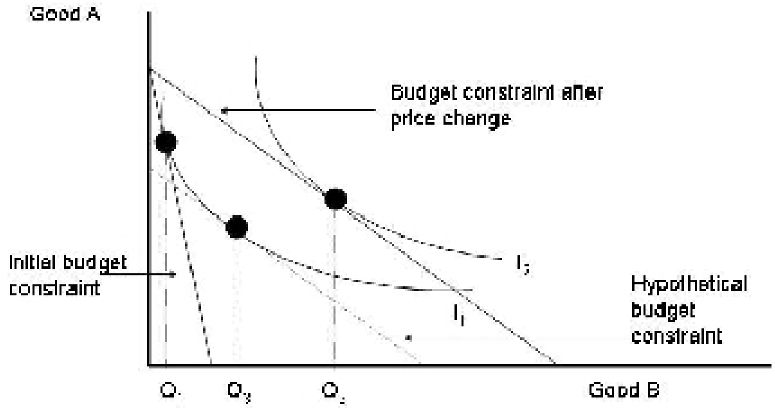



## 2 Effects of a price change

We know that a price change will cause the budget constraint to pivot on the intercept of the good for which price does not change. However, we can also decompose the effects of price changes into two pieces, the substitution effect and the income effect. The substitution effect is the part of the quantity purchased increase that occurs from the price of the good being now relatively lower (maintaining the same utility). To find the substitution effect we need to find the bundle that the consumer WOULD have bought had he faced the same relative prices (after the price change) but been forced to remain on his initial indifference curve. The income effect is the part of the quantity purchased increase that occurs from the consumer's income being expanded due to a relatively lower price. The two effects combined give us the total effect, which is the actual amount that the quantity purchased of a good increases or decreases by when the price of the good changes. Thus, we have:

$$Total\ effect = Substitution\ effect + Income\ effect$$

The easiest way to describe income and substitution effects is to look at how a consumer's optimal decision changes when the price of one of the goods changes. Consider a decrease in the price of good B, as shown on the graph below.



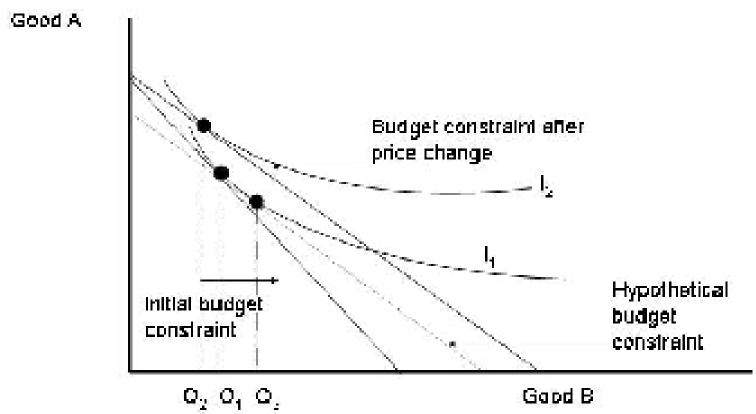

Now, to explain the graph. This consumer's initial optimal bundle was $Q_1$. After the price of Good B decreased his new optimal bundle became $Q_2$. Thus, the total effect is: $Q_2 - Q_1$. We now want to decompose this total effect into the substitution and income effects. Recall that the substitution effect is given by the amount by which quantity purchased would change IF the consumer had initially faced the relative prices of the goods after the price change (so if the slope of the budget constraint was the same as it is AFTER the prices change) AND he was held at his initial utility level (so we keep him on $I_1$). In order to find the substitution effect we simply shift the new (after the price change) budget constraint back until we find the optimal bundle that the consumer would have purchased on $I_1$ had he faced these relative prices. This is point $Q_3$ in the graph. The substitution effect is then given by $Q_3 - Q_1$, since that is the amount by which quantity purchased would increase if the consumer faced the same relative prices after the price change and was forced to remain on $I_1$. The income effect is then the remaining piece of the total effect, which in this case is $Q_2 - Q_3$. Notice that if we add the income and substitution effects we get:

$$Q_3 - Q_1 + Q_2 - Q_3 = Q_2 - Q_1$$

which is just the total effect.

## 2.1 Giffen goods

A Giffen good is a good that "disobeys" the Law of Demand. For Giffen goods, a decrease in the price of a good will actually cause LESS of the good to be purchased. We can use income and substitution effects to show that this is due to a negative income effect. The picture below uses indifference curve analysis and the decomposition of the total effect into substitution and income effects to show this.




Notice that the consumer purchases $Q_1$ when facing the initial prices. After the price of Good B falls, the consumer then purchases $Q_2$. Notice that the total effect in this instance, which is still $Q_2 - Q_1$, is negative because $Q_2 < Q_1$. To find the substitution effect we create the hypothetical budget constraint and find $Q_3$. Note that the substitution effect is still $Q_3 - Q_1$, which is still positive.[1] However, the income effect, which is still $Q_2 - Q_3$, is now negative. In the case of these particular goods and this price change, the negative income effect dominates the positive substitution effect, and the result is that as the price of Good B falls the consumer purchases less of that good.

The existence of such good is debatable, although anyone who wants to discuss the possibility of collectible items being Giffen goods is free to stop by my office.

## 2.2 Income and Substitution effects mathematically – the Slutsky equation

It is also possible to derive the income and substitution effects mathematically. In general, a demand function will be a function of prices (possibly of all goods)

[1] The substitution effect will ALWAYS be positive for price decreases, and will ALWAYS be negative for price increases.

and income, among other things. Assume it is a function of prices and income, so that demand for good $x$ is given by $x(p, y)$. Differentiating the demand function by the own-price of the good leads to:

$$\frac{dx(p,y)}{dp_x} = \frac{\partial x(p,y)}{\partial p_x} + \left(\frac{\partial x(p,y)}{\partial y}\right) * \left(\frac{\partial y}{\partial p_x}\right).$$

But from the budget constraint it can be seen that:

$$\begin{aligned} y &= x * p_x + z * p_z \\ \frac{\partial y}{\partial p_x} &= x \end{aligned}$$

so now we have:

$$\frac{dx(p,y)}{dp_x} = \frac{\partial x(p,y)}{\partial p_x} + x(p,y)\left(\frac{\partial x(p,y)}{\partial y}\right).$$

This is just decomposing the total effect into the substitution effect and the income effect. The term $\frac{\partial x(p,y)}{\partial p_x}$ is the substitution effect, while the term $x(p,y)\left(\frac{\partial x(p,y)}{\partial y}\right)$ is the income effect. Suppose we had the demand for good $x$ as

$$x(p,y) = \frac{\alpha y}{p_x}$$

where $\alpha \in (0,1)$. Then the substitution effect is:

$$\frac{\partial x(p,y)}{\partial p_x} = -\frac{\alpha y}{(p_x)^2}$$

and the income effect is:

$$x(p,y)\left(\frac{\partial x(p,y)}{\partial y}\right) = \frac{\alpha y}{p_x} * \frac{\alpha}{p_x} = \frac{\alpha^2 y}{(p_x)^2}.$$

The total effect is:

$$\frac{\alpha^2 y}{(p_x)^2} - \frac{\alpha y}{(p_x)^2} = \frac{\alpha y}{(p_x)^2}(\alpha - 1)$$

which is negative because $\alpha \in (0,1)$. Thus, when the price of this good increases, the quantity demanded will fall.

# 3 Expected Utility

What follows is a brief discussion of expected utility theory, which is one method that can be used to determine how individuals make decisions under uncertainty. There are some who claim that expected utility does not accurately describe how individuals make decisions under uncertainty and there is some evidence to support these alternative theories. However, expected utility is still the benchmark model which others are compared against, which is why we will discuss it.

Consider an individual who is considering working commission or working at a salary. Once working, either a good or bad outcome can occur. If on salary, the good outcome occurs 99% of the time and the individual earns \$1510, while the individual earns \$510 the other 1% of the time. With the commission job, the good outcome occurs 50% of the time and the individual earns \$2000 while the bad outcome occurs the other 50% of the time and the individual earns \$1000.

One way to compare risky alternatives is to look at their expected values, which is just the weighted sum of the outcomes. The expected value of the salary job is $0.99 * 1510 + 0.01 * 510 = 1500$. For the commission job, the expected value is $0.5 * 2000 + 0.5 * 1000 = 1500$. So neither job has a higher expected value than the other. It's also possible to compare them by variance, especially given that the expected value's are the same. The variance for the salary job is $0.99 * (1510 - 1500)^2 + 0.01 * (510 - 1500)^2 = 9900$, while the variance for the commission job is $0.5 * (2000 - 1500)^2 + 0.5 * (1000 - 1500)^2 = 250,000$. So the salary job has a much lower variance than the commission job, and given that they have the same expected value the individual might consider the one with the lower variance.

## 3.1 Risk attitude

Some people may like risk, some may be indifferent towards it, and others may dislike risk. Consider getting \$5 for certain or having a coin flip that pays \$10 if heads and \$0 if tails. Both have the same expected value of \$5, but the certain \$5 has lower variance than the coin flip. If an individual prefers the certain \$5 to the coin flip then the individual is said to be risk averse. If the individual is indifferent between the two options, then the individual is risk neutral. If an individual prefers the coin flip to the certain \$5 then the individual is risk loving. Note that an individual's risk attitude does not have to hold for every single decision – an individual who is risk loving in the example above may quickly become risk averse if the stakes are changed to \$1 million for certain against a coin flip of \$0 if tails and \$2 million if heads.

When decisions are made under uncertainty we refer to the individuals expected utility function. The expected utility function simply weights the utility of each outcome by its probability of occurring. Thus, the expected utility of a certain \$5 is $1 * u(5)$ or $u(5)$. The expected utility of the coin flip (assuming it is fair) is $0.5 * u(10) + 0.5 * u(0)$. Note that this is different than the expected value, which uses the actual amounts rather than the utilities of those amounts. If the individual's utility function is $u(x) = x$ then the expected value and the expected utility are the same. But what if the individual's utility function exhibits diminishing marginal utility (meaning that the more you have of one good, the less additional utility you receive from an additional unit – Bill Gates receives less utility from an additional dollar than a homeless person)? Then perhaps we have $u(x) = \sqrt{x}$. If this is the case, then $u(5) = \sqrt{5} = 2.2361$ and the utility of the coin flip is $0.5 * \sqrt{10} + 0.5 * \sqrt{0} = 1.5811$. Thus, an individual with utility function $u(x) = \sqrt{x}$ would prefer the certain \$5 to the coin flip, and would be risk averse.