Inadvertent Red Light Violations: An Economic Analysis

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Abstract

In recent years, several public policy initiatives have aimed at the increasingly common red light violation. Policies ranging from public service announcements, increased penalties for red light violations and, most strongly debated, the use of red light cameras to ensure near perfect enforcement of red light violations have been employed across the country. However, these initiatives have failed to reduce the number of red light violations to zero, a situation that has frustrated public policy-makers. This paper suggests that while some drivers may purposefully violate red lights, there are natural conditions under which drivers find themselves inadvertently running red lights. If drivers inadvertently run red lights, the traditional policy tools of enforcement and deterrence to combat illegal behavior will be less effective. We show that past public policy initiatives may have focused on an inappropriate source of red light violations, thereby having less success than proponents had hoped.

Keywords: Red lights, traffic control, public policy, illegal behavior.

JEL Classifications: D78, K42.

"The stop-and-go signs, garish ghosts in the sleet, went through their irrelevant tomfoolery again and again, telling the glacier of automobiles what to do. Green meant go, Red meant stop. Orange meant change and caution."
-Kurt Vonnegut, Jr. in *Cat's Cradle*, part 14.

1 Introduction

An increasingly frail observance of the standard red-light is a concern of many public-policy makers. Unable to affordably monitor and enforce driver behavior at red-light intersections using traditional techniques, policy makers have taken several different approaches to deal with red light violations, including alternative means of enforcement, e.g., cameras. As such, red light violations are implicitly assumed deliberate and voluntary. We question this assumption (and its implications) by investigating the conditions that can cause *inadvertent* red light violations. If many red light violations (hereafter "violations") are inadvertent, effective public policy may significantly differ from those currently suggested and widely supported.

The focus on violations has become more acute in recent years and may be appropriate considering the high social costs of this particular moving violation. According to the Federal Highway Administration, in 1999 there were 92,000 automobile crashes caused by running red lights in United States urban areas.[1] These accidents resulted in 90,000 injuries and 950 fatalities with estimated social costs, including property damage, injury, lost time and death, exceeding $7 billion (FHA, 1999). The same source cites surveys in which 55.8% of Americans admit to running red-lights and 99.6% of drivers fear being hit by another driver running a red light.[2]

There is relatively little agreement on what role the stop light serves. There seems to be two prevailing attitudes. One focuses on the possible efficiency and safety gains from signals that improve traffic flow on major roadways, thereby reducing commuting time, idle engines (a major source of air pollution), and perhaps driver frustration. The alternative views the stop light as a way to reduce average speeds that drivers can attain during the course of their trip. However, reducing average speeds may not reduce the variance of speed across drivers, a potential source of automobile accidents.

The sources of red light violations have not been investigated by economists, although the problem incorporates several economic tradeoffs and choices. This paper investigates the efficacy of red light cameras to reduce the number of drivers who enter or are in an intersection against a red light. This method of enforcement has proponents, who seek to reduce the social costs of red light running, and antagonists, who fear losses in civil liberties or an expansion of the state through technology.

The motivation for our study is the fact that perfect enforcement by red light cameras has not been successful at eliminating violations. We show that the problem of violations is not easily solved, whether by red light cameras or other popular policy choices. At the heart of the problem is that rational, law–abiding drivers can find themselves in a situation where they might inadvertently run a red light. This situation has been coined the "dilemma zone." The dilemma zone is influenced by many public policies, of which only one, and not the most effective, is the use of red light cameras. However, it is unambiguously clear that increasing the length of the amber phase decreases the dilemma zone, thereby reducing the probability that rational drivers inadvertently run a red light. Unfortunately, this policy has received relatively little support.

However, as we show, there are many influences on violations other than the length of the amber phase. As such, public policy-makers may do well by addressing any of these other issues. In particular, recent developments in so-called "traffic calming" initiatives, improved speed-limit enforcement, and improved road construction may be more appropriate public policy targets, vis-a-vis, red light cameras.

The paper is structured as follows. In Section 2 we outline the extent of red light violations and recent research and public policy directed towards the problem. In Section 3 we show that the utility maximizing velocity (speed) is likely greater than the actual velocity drivers can attain, and how various variables influence the optimal velocity for the representative driver. The following section investigates how the amber phase, as set by engineers, combined with optimal driver speeds naturally creates a driver's dilemma zone, in which cameras and other policy tools may be ineffective. In Section 5, we discuss the importance of our model in the context of possibly more efficient alternatives to reduce red light violations. The final section offers concluding remarks and suggestions for future research.

2 The Red Light Violation: An Overview of the Problem

Numerous engineering studies have investigated the nature of red light violations; most documenting the rate of violations at specific locations. For example, Deutsch, Sameth and Akinyemi (1980) found that 70% of all light cycles surveyed in Baltimore, Maryland, had at least one violation. Retting and Williams (1996) found a violation rate of 33.6% during an eight-week study in Arlington, Virginia, and that drivers who were younger, did not wear seat belts, had poor driving records and drove older cars were more likely to run red lights. Porter and England (2000) found a violation rate of 35.2% in several southeastern Virginia cities, and that cities with higher traffic volumes and larger intersections tend to have more violations. They also found that non-Caucasians and those who did not wear a seat belt were more prone to violations.

These studies indicate that there are certain characteristics about drivers and the intersections they approach that contribute to red light violations. However, it is admittedly difficult for those who observe red light violations to determine if the violation is intentional or not. A general, but perhaps not legal, definition for a red light violation is given in Retting and Greene (1997): "[t]he term 'red-light runners' is used ... to describe vehicles that entered [the intersection] on red *and does not necessarily imply that the drivers deliberately intended to run the red light*" (p.1) (emphasis added). Retting and Williams (1996) categorize intent by comparing the decision to drive through an intersection to the time elapsed since the red phase began. They find that the degree of violation, or intent, had little influence on their comparisons between those who complied with the red/amber phase and those who did not.

Many of these engineering studies prescribe some public policy to combat the red light running problem. Some advocate so-called "traffic calming" policies such as roundabouts (circles), intentional bends in roads, speed bumps, or ripple strips in an attempt to slow drivers when approaching an intersection. Others advocate increased levels of enforcement or increased minimum penalties for running red lights, in the spirit of Becker (1968). The proponents of increased enforcement admit that increasing human monitoring of red light violations is prohibitively expensive but that technological advancement now allows cameras to capture pictures of vehicles (and drivers) entering or being in an intersection after the red phase begins, thereby increasing enforcement efficiency while reducing average enforcement costs.

Legislation allowing the use of this technology has been implemented in a number of states. As of August, 2000, ten states and the District of Columbia had enacted legislation, three states had legislation pending and eight others had legislation under consideration. In 1990, no jurisdiction employed camera enforcement; by 2000, more than 70 jurisdictions employed the technology.[3] While a recent survey indicated that between 76 and 80 percent of drivers interviewed favored cameras, many policy advocates argue against cameras on privacy and civil liberty concerns.

In May 2001, the office of House Majority Leader Dick Armey released a report (hereafter the 'Armey Report') purporting that amber phases have been reduced in many U.S. cities so to increase the number of violations. The Report further questions the use of red light cameras, arguing that the costs in terms of civil liberties outweigh the benefits of cameras (in reduced red light violations).

However, proponents of camera enforcement cite a correlation between cameras and a reduction in red light violations; for example, after installing cameras at eighteen intersections, New York City experienced a sixty-two percent decline in violations between 1994 and 1996. However, whenever cameras are installed the number of violations falls to some positive steady state, indicating that even perfect enforcement does not reduce the number of violations to zero. In response, many public officials have switched to increasing the penalty for running red lights. In 1998 California increased the minimum fine for a violation from $35 to $270, yet even this dramatic increase has not significantly reduced violations.

Another common concern about the use of cameras is the net costs to a locality by using this enforcement mechanism. Many cities have found that red light cameras are initially profit earners (for the locality) but that, as drivers adjust to the new enforcement mechanism, cameras often become money losers for local governments.[4] The price of cameras installed in Charlotte, North Carolina, was $50,000 each with another $50,000 in expected yearly operation costs.[5] Installed in 1998 and maintained and operated by Lockheed Martin, each $50 citation yielded $14 for the city and $36 for Lockheed Martin (Whitacre, 1998). However, while in June of 1998 a test camera captured 17 violations each hour, by August of that year the camera captured only six violations per hour.

The Armey Report suggests that many cities, in response to the reduction in captured red light violations, reduce the length of the amber phase, therefore causing more red light violations and an increase in the revenue generated by red light cameras. Indeed, as a Lockheed Martin spokesman admitted, "We are not pleased with what we're getting ... and we're absolutely confident the numbers will get better [for Lockheed Martin?]" (Whitacre, 1998), although whether better numbers through improved camera technology or through reduction in amber phase durations was not specified.

Clearly those who operate red light camera systems are concerned about their ability to make a normal return on the systems after the immediate impact of the cameras on violations. Thus public policy, e.g., amber phase durations, may be determined based on private concerns rather than the public good. Indeed, in San Diego a County Superior Court judge dismissed almost three hundred citations captured by cameras because of the private gains to the firm managing the camera system (Fox, 2001). The problem of disparate incentives might be solved if cities undertake the management of camera systems, although the potential for less obvious principal-agent problems, as described in the Armey Report, would still exist.

While many argue against red light cameras for civil liberties concerns, we show that such enforcement mechanisms are not as effective in reducing the natural problem of violations as other, more direct, traffic control policies. In the model we develop below, we do not argue that cameras and other enforcement mechanisms do not impact voluntary, deliberate red light violations. Rather, we focus on the potential for inadvertent violations and how previous studies of red light violations have ignored crucial elements of the problem.

3 A Representative Driver Model

In analyzing the red light violation we must first consider the actions of the representative driver assumed to maximize utility, $\nu_i = U(C_i, \ell_i)$, concave and twice continuously differentiable in both arguments: consumption, C_i, and leisure, ℓ_i. Utility is maximized subject to a time constraint and budget constraint, the latter containing both the cost of consumption and the expected costs associated with running a red-light.

The driver's budget constraint includes the perceived cost of a red-light. Define T as (actual) time spent at the intersection under a red-light. Let the perceived cost of waiting at the intersection is terms of wages lost and expected time at the light be $W_i T^e$, where $T_i^e = T + \epsilon_i$, $\epsilon_i \sim N(0, \sigma_i^2)$, is the *expected* total time spent waiting at the intersection, and W_i, is a competitively determined wage rate.

Instead of stopping when confronted with an amber phase, the representative driver must decide whether or not she can safely make it through the intersection given her expected costs of waiting. However, in doing so she may incur the expected cost of running a red light, which is comprised of two elements. First, is the expected cost of fines incurred by the violation, which is assumed to be a function of driver i's velocity, V_i, given by

$$\pi^e(V_i)F, \tag{1}$$

where $\pi_V > 0, \pi_{VV} > 0$, and $\pi^e(V_i)$ is the subjective probability of incurring a fine and F is the level of the fine, assumed the same for all drivers.

The expected cost of a crash is defined as the product of the driver's subjective probability of a crash, φ^e, and a damage function. Damage is assumed to be a function of the driver's velocity, her wage, and the wage of the other driver involved in the accident, W_{-i}, or

$$D(V_i, W_i, W_{-i}), \tag{2}$$

where $D_V > 0,\ D_{W_i} > 0,\ D_{W_{-i}} > 0,\ D_{VW_i} > 0,\ D_{VW_{-i}} > 0$ Combining the cost equations (1) and 2, we obtain the total expected cost of running a red-light is given as

$$RC = \varphi^e D(V_i, W_i, W_{-i}) + \pi^e(V_i)F - W_i T^e.$$

Each individual earns a wage, W_i, for selling labor $L_i = \lambda_i \ell_i$, where $\lambda_i > 0$ is the percentage of leisure time spent at work, and consumes C_i units of a composite consumption good at price P. In addition, i's income must pay for any traffic violations that arise if caught running a red light and/or causing an accident. Therefore, the budget constraint can be written as

$$W_i(\lambda_i \ell_i + T^e) = PC_i + \varphi^e D(V_i, W_i, W_{-i}) + \pi^e(V_i)F. \tag{3}$$

The time spent waiting at a red-light can be interpreted as a "lump-sum tax" and implies as wages go down the perceived cost of running a red light decreases.[6]

A time constraint is also faced by the representative driver. An individual's time is spent in leisure, labor, and travelling to and from work, i.e., commuting. While trip time may seem a small part of the work day, according to the *1990 Census of Population Report* the average one way commute over a sample of 436 cities was 22.4 minutes (Census, 1990); with a 50 week work year, commute time almost equals the standard two week vacation. For workers with a commute of more than 45 minutes, average commute is about one hour, equivalent to *ten* work weeks over the course of a year.

To formalize the concept, let commute time be a function of velocity, expected time at the intersection, T^e, and road congestion, Q, i.e., $t(V_i, Q, T^e)$, with $t_V < 0, t_Q > 0, t_T > 0, t_{VV} > 0, t_{QQ} < 0, t_{TT} < 0$, and $t(\cdot)$ assumed identical across all drivers. Congestion is a function of: (i) increased traffic;[7] (ii) road capacity; (iii) the velocity of drivers other than i; and/or (iv) policy driven intentional congestion caused by traffic calming devices.[8] Normalizing time to 1, we can write i's time constraint as $1 = (1 + \lambda_i)\ell_i + t(V_i, Q, T^e)$ or

$$\ell_i = \frac{1 - t(V_i, Q, T^e)}{(1 + \lambda_i)}, \tag{4}$$

a variation of the time constraint in Owen (1969).

The Driver's Problem

The problem for driver i is to maximize $U(C_i, \ell_i)$ subject to the constraints (3) and (4) which yields the first order conditions (FOCs):

$$\begin{aligned} C_i: &\quad U_C - \mu P = 0, \\ \lambda_i: &\quad U_\ell \ell_\lambda + \mu W_i[\ell + \lambda \ell_{\lambda_i}] = 0, \\ V_i: &\quad U_\ell \ell_V + \mu[W_i \lambda_i \ell_V - \varphi^e D_V - \pi_V F] = 0, \end{aligned}$$

where μ is the Lagrangian multiplier (the shadow price of income). Manipulating the first two FOCs yields the standard indifference relation: $U_\ell/U_C = W_i/P$. Combining this with the last FOC implicitly characterizes i's optimal velocity, $V_i^*(T^e, \varphi^e, \pi^e, F, \lambda_i, W_i, W_{-i}, Q)$, which yields

$$W_i \ell_V(V_i^*)(1 + \lambda_i) = \varphi^e D_V(V_i^*) + \pi_V(V_i^*)F. \tag{5}$$

While this equation appears unfamiliar, the economic intuition is straightforward. The left side of equation (5) represents i's marginal value of velocity and the right side is the expected marginal cost. The term $\ell_V(V_i^*)(1 + \lambda_i)$ is the additional leisure and labor time when speed increases; this multiplied by the nominal wage yields the marginal value of speed. The marginal cost of velocity, $\varphi^e D_V(V_i^*) + \pi_V(V_i^*)F$, represents the expected cost of an increase in speed in terms of potential fines accrued and the potential accident damage incurred at a higher velocity. Hereafter, the i subscript will be dropped unless necessary, though we continue to use the $-i$ subscript.

Comparative Statics

It is of interest how optimal velocity, V_i^*, changes with $W, \lambda, Q, W_{-i}, \varphi^e$, and the standard policy targets, π^e and F. The comparative statics are:

$$\begin{aligned} W: &\quad -\vartheta(t_V + \varphi^e D_{VW}) < 0 \\ Q: &\quad -\vartheta W t_{VQ} > 0 \\ T^e: &\quad -\vartheta W t_{VT} > 0 \end{aligned}$$

$$
\begin{aligned}
W_{-i}: &\quad \vartheta\varphi^e D_{VW_{-i}} > 0 \\
\lambda: &\quad 0 \\
\varphi^e: &\quad -\vartheta D_V < 0 \\
\pi^e: &\quad -\vartheta F \pi_V < 0, \\
F: &\quad -\vartheta \pi_V < 0,
\end{aligned} \tag{6}
$$

$\vartheta \equiv (Wt_{VV} + \varphi^e D_{VW} + \pi_{VV} F)^{-1} > 0$.

With respect to driver i's wage we get an intuitively appealing result. Higher wages increase the opportunity cost of being involved in an accident both in terms of lost wages and higher penalties associated with repairs to vehicle and body and therefore lowers the utility maximizing velocity, *ceteris paribus*. For unemployed drivers $W = D_{VW} = 0$ and velocity increases vis-á-vis employed drivers because the expected costs of speeding are relatively lower.[9]

Consider the effects of changes in the conditional probabilities of enforcement or accident on the impact of wages on velocity. These probabilities are often the target of public policy initiatives, such as red light cameras. It is clear that both $V^*_{W\varphi^e} < 0$ and $V^*_{W\pi^e} < 0$, but note that these changes are only second order and to dramatically reduce velocity, given a change in wages, either the level of the fine must become increasingly high, $F \to \infty$, and/or $\pi^e = 1$, i.e., red light cameras.

Traffic congestion has a positive impact on optimal velocity which may initially seem counterintuitive because higher congestion will tend to reduce the *actual* velocity attained, say V^α. However, recall that V^* is the utility maximizing velocity. Therefore, congestion increases time spent travelling which motivates drivers to reduce travel time by increasing optimal velocity, i.e., $V^* > V^\alpha$. This helps explain aggressive driving, often manifested in "darting" in and out of lanes on highly congested roads.[10] Likewise for expected wait time at the intersection; drivers who anticipate a long wait increase their velocity to avoid being caught at the light.

The sign of $-i$'s wage suggests, via the damage function, an individual, in hopes of getting the payoff from litigation and insurance, would prefer to have an accident with a person with a relatively high income. Optimal velocity does not change with changes in λ because labor yields consumption and both consumption and leisure are 'goods': the leisure-labor choice has no bearing on optimal velocity. Finally, velocity decreases as φ^e, π^e and F increase, but these decreases are limited by the marginal impact of velocity on the damage function and the probability of being penalized, respectively.

4 The Dilemma Zone: A Source of Inadvertent Red Light Violations

The red-light-running literature is divided into two strains. One primarily focuses on the demographics and intersection characteristics that correlate with or encourage red light violations. The other deals with engineering factors in signal timing and intersection construction. This latter strain typically discusses what is termed the dilemma zone (DZ), which is defined as the distance from the intersection where the driver does not have time to avoid being in the intersection when she stops nor to safely traverse the intersection before the cross-traffic has a green light. Obviously a dilemma zone introduces the possibility that drivers may inadvertently run red lights because of physical laws, perhaps causing unintended collisions and damage. As such, public policy might arguably focus on attempts to reduce the dilemma zone for the average driver.

Unfortunately, the literature on the engineering aspects of signal timing ignores the fact that the position and shape/size of the dilemma zone are unknown to both public designers and the driver. An important impact of this omission is that the driver must guesstimate the location of the dilemma zone at every intersection she encounters. Any error in the determination of the safe distance to stop, or a driver caught in the dilemma zone at the time the amber phase begins, manifests itself as a driver caught in an intersection during the cross traffic's green phase. Understanding the nature of the dilemma zone for given driving characteristics and driver velocities is a requisite to discerning the causes for red light violations and any attempts to formulate informed public policy to combat these violations.

We augment a simple model which describes the relationship between the width of the intersection; the ideal (or engineer's) velocity, and the length of the amber phase; the length of the vehicle; and the driving characteristics of the vehicle and driver (acceleration, deceleration) formulated by Gazis, Herman, and Maradudin (1960, hereafter GHM) to describe the DZ and ideal versus individual driving patterns. The GHM model takes velocity and driving characteristics as given and then derives an "optimal" amber phase and defines the DZ under perfect information.[11] Unfortunately, driver behavior does not conform to the expectations of the traffic engineer. Above, we show driver velocity is the result of utility maximization and likely deviates from that chosen by the engineer, who anticipates drivers follow the posted speed limit. In a footnote the Institute of Transportation Engineers states in an Informational Report that "...engineering judgement should be utilized in the timing of vehicle signal change intervals" (Institute of Transportation Engineers, 1994, p.9). This statement implicity ignores the economic decisions of drivers and assumes that all drivers conform the engineer's objective function and parameters.

To assist in the discussion, which is not explicitly economic in nature, we define the following variables:

1. V_0: Engineer's optimal velocity (which we assume to be the speed limit);
2. X : the distance from the intersection;
3. s_{i1}/s_{i2}: driver i's reaction time to accelerate/decelerate;
4. a_1/a_2: constant of acceleration/deceleration;
5. ω^e: i's believed/expected width of the intersection ($\omega^e = \omega + \epsilon_\omega$, where ω is actual width of the intersection and $\epsilon_\omega \sim N(0, \sigma_\omega^2)$);
6. Λ_0: length of the average vehicle.
7. T_A^e: the expected length of the amber signal ($T_A^e = T_A + \epsilon_A,\ \epsilon_A \sim N(0, \sigma_A^2)$).

Item 4 represents vehicle characteristics and, for simplicity, is assumed identical across all drivers/vehicles, ϵ_ω represents driver i's idiosyncratic mistake about the width of the intersection, and ϵ_A represents driver i's mistake about the length of the amber phase.

The Amber Phase Interval

Previous analyses have focused primarily on the physical construction of intersections and, more particularly, the timing of green, amber and red phases. We restrict our focus on the amber and red phases of the light. Almost universally, the analysis is performed by traffic engineers assuming that drivers are atomistic agents with perfect information and foresight, homogeneous driving habits and instantaneous reaction times. As we shall see, such assumptions are rather limiting and fail to account for much of what economists have long recognized about human behavior.

There is no universal policy for determining the length of the amber phase, although there are several suggested practices. The *Manual on Uniform Traffic Control Devices,* published by the U.S. Department of Transportation, recommends amber phases between three and six seconds. Another set of guidelines is published by the Institute of Transportation Engineers in the *Transportation and Traffic Engineering Handbook* (1999). While recommended amber phase durations have changed slightly over time, the standards are based upon approach speed, stopping ability, driver response time, intersection width and the length of the car. Of these parameters, only one is fixed at the time the amber phase is determined, the intersection width. All other parameters vary from car to car and driver to driver. It is apparent that the social planner selects averages of the parameters from the local driver characteristics in order to determine the length of the amber phase.

The standard amber phase is selected such that a driver approaching the intersection at the posted speed limit and driving an average vehicle, when at a critical distance from the intersection when the amber phase begins, can either just stop her vehicle or continue at a constant velocity safely through the intersection; see Figure 1, where distance, X, is measured along the vertical axis and time is on the horizontal axis.[12] The solid diagonal represents the velocity V_0 and allows the driver to safely traverse the intersection. Alternatively, the driver could decelerate to a stop, which is represented by the dotted curve. The guidelines suggest that if the adequate amber phase is greater than five seconds, the phase should be set at five seconds followed by an all-red phase of two to three seconds to allow vehicles in the intersection time to safely exit before cross traffic begins.

This approach is adequate for a driver with perfect information in the following ways. If the vehicle is closer to the intersection than the critical distance, the driver should continue through the intersection even though the light is amber. An attempt to stop will cause the vehicle to come to rest in the intersection. On the other hand, if the vehicle is further from the intersection than the critical distance then the driver should stop because an attempt to cross the intersection at a constant speed will not be successful. We note that in Figure 1, the dilemma zone is a single point in space, and therefore the dilemma zone is effectively non-existent, i.e., no inadvertent red light violations (and accidents) occur.

This ideal amber phase can be quantified as $T_A = V_0/2a_2 + (\omega + \Lambda_0)/V_0$, where V_0 is the posted speed limit, a_2 is the average constant deceleration constant, ω is the width of the intersection and Λ_0 is the average length of a car. The first term is the time required to stop the vehicle travelling at V_0, with driver reaction time set to zero, and the second term is the amount of time for the vehicle to traverse the intersection.

While this formula generates amber phase durations according to the method proposed by GHM (1960), it most assuredly understates the true amount of time the amber phase should be set. First, the assumption that the approach speed is the posted speed limit is most likely inappropriate. Second, the amber duration fails to account for two critical elements, even if drivers have perfect information and foresight: the time it takes for the driver to decide whether to stop or not and the distance that must be travelled to the intersection if the decision to run the intersection is made.

The Dilemma Zone Defined

The driver is assumed to have imperfect information about the variables known to the engineer: the width of the intersection, and the length of the amber phase. The optimal speed, determined in equation (5), is assumed fixed, given the parameters previously discussed. The driver thus faces two choices when confronting an intersection during the amber phase: (*i*) stop or (*ii*) continue through the intersection. Each of these choices has a limiting condition.

The first, in the case of stopping, is given by $X - Vs_{i2} \geq V^2/2a_2$, where X is the distance the driver is from the intersection when the amber phase begins. If the distance required to stop is less than the distance from the intersection, the driver stops. This is represented by the dotted line in Figure 1. Given this condition, there exists some rate of deceleration, say a_2^*, such that the driver's vehicle travelling V^* can safely and *comfortably* come to a stop, that is without skidding or losing control. The rate of deceleration from speed V^* yields the "critical distance"

$$X_C^* = V^* s_{i2} + \frac{(V^*)^2}{2a_2^*}. \tag{7}$$

For any distance $X > X_C^*$ there is no dilemma and the driver can safely stop independent of the length of the amber light, T_A, given the reaction time s_{i2}. However, if $X < X_C^*$ the driver must decide whether to attempt to stop.

The second condition is whether the speed limit, V_0, is sufficient to safely traverse the intersection, that is if

$$X + \omega + \Lambda_0 - V_0 s_{i1} \leq V_0(T_A - s_{i1}) + \frac{1}{2}a_1(T_A - s_{i1})^2,$$

where $X + \omega + \Lambda_0$ is the total distance required to safely exit the intersection (the diagonal solid line in Figure 1), *without* accelerating, $a_1 = 0$. The amber phase duration required to safely cross and exit the intersection can be written as $T_A = (X + \omega + \Lambda_0)/V_0$, or

$$X_0 = X_A - (\omega + \Lambda_0), \tag{8}$$

where $X_A = V_0 T_A$ is the distance travelled during the length of the amber phase at velocity V_0. This is the *fixed* minimum distance that can be traversed without having to accelerate given the predetermined speed limit, the length of the amber signal, and the distance $\omega + \Lambda_0$. While this well defined distance may appear a theoretical abstraction, such exogenous predetermined distances can be found in practical use. Many municipalities use road markings to signal drivers the location of X_0 given the length of the amber light, speed limit, and intersection width.[13] This distance is less than the critical distance and the relationship between the two is shown in Figure 2.

We can now characterize the DZ. Without loss of generality, normalizing $V_0 = 1$, and noting that $T_A^e = T_A + \epsilon_A$ and $\omega^e = \omega + \epsilon_\omega$, subtracting equation (8) from (7) yields the *width* of the DZ given velocity V_i^*,

$$\widehat{DZ}_i = s_{i2}V_i^* + \frac{(V_i^*)^2}{2a_2^*} - (X_A^e - \omega^e) + (\epsilon_A - \epsilon_\omega) + \Lambda_0. \tag{9}$$

No problems arise for the driver when the amber phase starts and her distance from the intersection is greater than X_C^* or less than X_0, the driver stops or continues safely, respectively. However, mistakes can be made when the driver is within the distance $\widehat{DZ}$ when the amber phase begins; she finds herself in the dilemma zone (the shaded area in Figure 2). While being in the DZ does not guarantee the driver will run a red light, it is in this region where unintentional mistakes are made and one consequence of these errors is an increased likelihood of violating a red light and/or being involved in an accident.[14]

Note that even under perfect information the dilemma zone is nonzero. Passing the expectation operator through equation (9) we have:

$$E(\widehat{DZ_i}) = V_i^* E(s_{i2}) + \frac{(V_i^*)^2}{2a_2^*} - (X_A - \omega) + \Lambda_0. \tag{10}$$

The variables X_A and ω represent the mean distance that can be traversed in time T_A and the mean intersection width, respectively. However, the variance of the DZ is a potentially important measure that no engineering treatment of red light violations has considered and is nonzero even if $V_i = V_0$ for all drivers. A higher variance in $\widehat{DZ}$ corresponds with a greater potential number of inadvertent violations.

Yet, the engineer's ideal DZ assumes drivers have homogeneous velocity $V_i^* = V_0$, and is simply a line at $X_C^* = X_0$, implying no inadvertent red-light running (and no accidents). Yet, if optimal conditions are not met, the DZ expands, possibly placing drivers in an unenviable situation. Moreover, an expanding DZ with a higher variance across drivers is a doubly unattractive alternative, N different dilemma zones. If the probability of being at any given point in space is equal and constant, the larger the DZ, the greater the probability of being in the DZ when the amber phase begins. Changes in $\widehat{DZ_i}$ indicate shifts in the dilemma zone closer to or further from the intersection. Many variables affect the DZ, increasing driver confusion over its location when approaching an intersection thereby increasing the probability of an inadvertent red light violation.

We first investigate the consequences of changes in the optimal velocity, vehicle and road characteristics on the DZ by once again deriving comparative statics. We obtain:

$$\frac{\partial \widehat{DZ_i}}{\partial V_i^*} = s_{i2} + \frac{V_i^*}{a_2^*} > 0, \tag{11}$$

$$\frac{\partial \widehat{DZ_i}}{\partial s_{i2}} = V_i^* > 0, \tag{12}$$

$$\frac{\partial \widehat{DZ_i}}{\partial a_2} = -\left(\frac{1}{2a_2^2}\right)^2 < 0. \quad (13)$$

Ultimately, the effects of the optimal velocity on the DZ found in equation (11) are of the greatest interest as it is via optimal velocity that driver preferences are manifested. This equation suggests that higher speeds are directly correlated with larger dilemma zones and may result in a greater number of involuntarily violations and an increased potential for more accidents. As driver velocity differs from that assumed by the traffic engineer, the difference between the actual DZ of the driver and the ideal DZ of the engineer also increases, a result of the engineer setting signal phases based on parameters different than those exhibited by many drivers.

Yet, if engineers fail to adjust to drivers' preferences manifested in their utility maximizing velocities, do other policies adjust driver choices to coincide with engineers' parameters? This seems difficult to accomplish. Red light cameras, which increase the odds of being caught or increasing fines for red light violations only have an indirect impact on driver velocities, thereby having only a second-order impact on the dilemma zone that drivers encounter. This helps explain why these policies fail to have the impact that proponents desire.

With respect to the reaction time of drivers deciding to decelerate, equation (12), the result is intuitively attractive. As reaction time to decelerate increases, so too must the area in which the decision is made increase. Conversely, driving with a higher deceleration coefficient, a_2, e.g. a BMW, the DZ shrinks relative to driving with a lower deceleration coefficient, e.g., a Pinto, because the time to make a decision to stop is less crucial, as shown in equation (13).

The Optimizing Driver and the Dilemma Zone

We now combine the results from the optimizing driver and the dilemma zone. The comparative statics in (6) and (11) are used for inference on driver behavior as she enters an intersection. Because the sign of equation (11) is positive we need only consider the signs from the driver optimization comparative statics.

We first consider how the driver's dilemma zone is affected by various parameters. As mentioned earlier, the width of the intersection is the only parameter known to the engineer when the length of the amber phase is set; all other variables change from driver to driver. This implies that each driver, regardless of their attempts to replicate the engineer's assumptions, faces a different dilemma zone.

The DZ increases with congestion, expected wait time, and W_{-i}. Moreover, the fine incurred for running a red light, π^e, contributes to the collapse of the DZ. Consider $\pi^e \to 1$, e.g. getting caught by a camera is a certainty, the DZ shrinks, but it will not collapse to zero because there is an error associated with location and depth of the DZ, as characterized by equation (9). Therefore, involuntary violations may likely continue.

Table 1 shows how the dilemma zone changes for a hypothetical driver with different approach speeds. The amber phase is selected according to the standards published in the 1999 Transportation and Traffic Engineering Handbook, assuming a 40 mph (66 feet/second) approach speed, a deceleration constant of 10 feet/second^2, a car length of 20 ft and an intersection width of 50 feet. With these parameters and published guidelines for engineers, the traffic engineer is "instructed" to set the amber phase at 4.36 seconds.

We calculate the dilemma zone for a hypothetical driver who matches the engineer's assumptions but has a reaction time of one second. This decision time naturally introduces a dilemma zone to all drivers, even those who match the engineer's assumptions; as the driver is making her decisions, she is still covering ground.

Looking at Table 1, as approach speed increases the dilemma zone likewise increases in width. An approach speed of 50 mph causes more than an 100% increase in the width of the dilemma zone. For speeds above 40 mph the time spent in the dilemma zone is greater than the reaction time. By definition, the dilemma zone is an area in which the driver cannot stop safely nor cross the intersection successfully. In these instances, the driver will decide to stop or not after one second and the decision is guaranteed to fail, although not guaranteed to be dangerous. Indeed, the driver can find herself in the dilemma zone after having made the decision to stop or attempt to cross the intersection.

Tables 2-5 replicate the calculations described above for various parameter values. Table 2 changes the reaction time of the driver from one second to one half of a second. The fourth column of Table 2 indicates that at each approach speed, the dilemma zone declines dramatically. A decrease in the dilemma zone reduces the probability that a driver will be in the dilemma zone when the amber phase begins, thereby reducing the probability of an involuntary violation. Table 3 changes the deceleration constant from the engineer's assumed deceleration constant. The fourth column of Table 3 indicates that for the first three approach speeds listed, the dilemma zone does not exist, thereby precluding any inadvertent violations. With the increased ability to stop, the dilemma zone is still shorter than the assumed length of the vehicle even when velocity is ten miles per hour over the speed limit. Table 4 changes the length of the vehicle, which was originally assumed to be 20 feet and is increased to 23 feet, e.g., a sedan versus a sport utility vehicle. In this case, compared to Table 1, the dilemma zone increases at all approach speeds.

Finally, Table 5 presents simulation results in which all parameters match the engineer but the amber phase is exogenously reduced by 0.36 seconds, again it is clear that the dilemma zone increases. Most disconcerting, however, is that a vehicle approaching the intersection at the posted speed limit and satisfying the engineer's parameters perfectly will experience an increase in their dilemma zone of almost 36%. If the probability of being in any particular position in relation to the roadway is equal, this small decrease in the amber phase can cause a dramatic increase in the number of inadvertent red light violations.

As approach speed increases, the distance back from the true dilemma zone at which the driver will make a bad decision in the dilemma zone is increasing. This phenomenon may help explain the prevalence of red light violations – decisions are being made during a time interval which the engineer does not take into consideration. While the remedy to this situation is not immediately clear, we outline possible solutions below.

5 Discussion

The previous section shows that the dilemma zone facing individual drivers can move away from the dilemma zone envisioned by the traffic engineer, but the dilemma zone can also expand and contract based upon the physical characteristics of a driver's car and her reaction time. Therefore it is difficult for the traffic engineer to solve the problem of inadvertent red light violations. As mentioned earlier, we can assume that the individual driver is able to attain an actual velocity less than or equal to her utility maximizing velocity, i.e., $V^{\alpha} = \Gamma(Q, V_0)V^*$, where V_0 is the legal speed limit, $\Gamma(\cdot) \leq 1$, $\Gamma_Q < 0$ and $\Gamma_{V_0} > 0$. Our formulation of $\Gamma(\cdot)$ implies the reduction from optimal velocity is caused only by physical variables exogenous to the driver's decision process. This has potentially dramatic implications for various public policy initiatives focusing on red light violations.

For example, the use of red light cameras to enforce and penalize red light violations is likely to have only a minimal impact on the number of *involuntary* red light violations. This is because the probability of enforcement and the penalties associated with enforcement only impact the optimal velocity of the representative driver, thereby having only an indirect impact on the actual velocity attained. Because approach velocity is one of the most important elements in determining the location and width of the driver's dilemma zone, policies that only indirectly impact a driver's velocity, such as red light cameras, will have less impact on the width and location of the driver's dilemma zone than policies affecting the actual velocity that drivers can attain, e.g., speed bumps and roundabouts.

This argument favors so-called "traffic calming" policies that reduce the actual velocity that drivers can attain, if the goal is to reduce the number of inadvertent violations.[15] Alternatives such as red light cameras will be effective, to a point, in generating local revenues through enforcement, but will have little impact on the number of inadvertent red light violations. Indeed, municipalities that install red light cameras notice an immediate decline in the total number of red light violations, but eventually obtain a non-zero steady state. It is possible that the majority of red light violations at intersections with red light cameras are inadvertent.

An alternative policy choice, as mentioned in the Armey Report of May 2001, is to extend the length of the amber phase, thereby reducing the size of the dilemma zone. Indeed, Retting and Greene (1997) found a statistically significant and substantial decrease in the number of red light violations when the amber phase is set *longer* than the length recommended by the Institute of

Transportation Engineers (ITE). Conversely, they found a dramatic increase in red light violations when amber phases were set below that recommended by the ITE. The results in this paper indicate that perhaps the amber phase is one of the most powerful tools to reduce the rate of inadvertent violations. These conclusions are supported by Wortman, et al. (1985) and Stimpson, et al. (1980) who found that extended amber phases reduced the number of violations and potential collisions, respectively.

Whether cities consciously reduce amber phases to increase the number of red light violations is still an empirical question. However, the Armey Report of May 2001, provides anecdotal evidence that many cities are using reduced amber phases to increase local revenues and, perhaps, to pay for the maintenance and operation of red light camera systems. Regardless of a locality's revenue needs, the use of reduced amber phases to purposefully increase the number of red light violations is an extremely dangerous public policy; one that deserves to be scrutinized and reformed.

Future empirical work should focus on several straight-forward hypotheses that follow from our analysis of red light violations:

1. Just as a driver's wage tends to slow him down, so too does it make the driver more cautious when entering an intersection. Wages might also capture attitudes towards risk. Evidence suggests that younger and less educated (and perhaps lower income), drivers tend to take more risks while driving. These two effects imply that higher income localities should experience fewer violations;

2. Externalities caused by congestion expand the dilemma zone, as does the expected length of the red light. Localities with high congestion and long signal phases should see more red light running;

3. A higher probability of being caught running a red light will tend to have a relatively minor impact on red light violations and intersection accidents by causing a minor reduction in the dilemma zone. Localities with red light cameras should see a slight decrease in red-light violations;

4. An exogenous decrease in amber phase durations, if unannounced to drivers so that they can

update their expectations about their dilemma zone, will cause at least a short-run increase in red light violations. While this may augment local revenues through increased fines, it is a very dangerous policy because of the increased likelihood of accidents; and

5. An increase in transient drivers will increase the variance of errors in expected amber phases and intersection widths thereby increasing the variance in the DZ across drivers. Therefore, localities with more transient drivers should have more violations.

6 Conclusions

Previous studies of red light violations have failed to recognize that the dilemma zone is likely to vary for each driver. Engineering treatments of red light violations assume drivers are atomistic agents who fully conform to the engineer's parameters. Moreover, red light violations are implicitly assumed deliberate and voluntary. If true, increasing enforcement and penalty levels may have a direct impact on the number of voluntary red light violations, consistent with Becker (1968). However, municipalities that install red light cameras see a decline in the number of violations to a non-zero steady state. The model we present in this paper helps explain why it is difficult to use enforcement mechanisms alone to "solve" the red light running crisis.

We show that it is virtually impossible to reduce the dilemma zone to zero, a condition that would remove all possibility of inadvertent red light violations. Several public policy initiatives can influence the dilemma zone and reduce the probability of inadvertent red light violations, however the policies that are most often debated, e.g., increased fines and enforcement mechanisms, have only an indirect, second-order effect on the location and width of a driver's dilemma zone, thereby having minimal impact on *inadvertent* red light violations.

We do not investigate whether city governments purposefully reduce amber phases to increase the number of red light violations, as purported in the Armey Report. Yet, it is clear from our analysis that reducing the amber phase is an effective way to increase the number of (inadvertent) red light violations, which may increase the revenues generated by camera systems. However, increases in local revenue must be weighed against the potential property and human damages incurred by red light violations. Creating a situation in which drivers are forced by physical laws to violate man-made laws seems counter to efficient public policy,

Rather than focusing on civil liberties as in the Armey Report, we argue against cameras because they are less effective in reducing inadvertent red light violations than alternative, perhaps less expensive, policies. These alternatives would target the velocity that individuals can attain.

Notes

[1]One should take statistics on red light violations in context. As the issue of red light running has become more politicized, statistics presented have become more confusing. The FHA numbers reported here have been enlarged by many other authors to include all intersection accidents (approximately 1.8 million in 1999) and deaths, regardless of whether a violation has taken place or not (see, for example, Retting, et al., 1998)

[2]These data can be found at http://safety.fhwa.dot.gov/.

[3]Cities such as New York, San Francisco, Los Angeles, Charlotte and Fairfax, Virginia, have implemented such enforcement mechanisms.

[4]According to the Armey Report, the revenues generated by cameras can be substantial. For instance, a single camera in Washington, D.C. reportedly collected more than a million dollars in fines. Cameras in San Diego collected more than $30 million over a recent eighteen month span.

[5]These numbers are hard to confirm. A 1998 report by Sunnyvale, California, estimated installation costs for a red light camera of $80,000 to $100,000. Estimated yearly operating costs for a system of five intersections with cameras are $330,000, including salaries and benefits for two employees capable of maintaining the system (City of Sunnyvale, California, 1998).

[6]This formulation differs from Owen (1969) who suggests that the value of travel time differs from that of time spent in work or leisure.

[7]Over 58% of the U.S. population now lives in the 35 largest MSAs.

[8]Thus, driving down a single-lane country road behind a single piece of slow moving farm equipment is considered road congestion.

[9]While this result may seem counter intuitive at first, the employed driver slows down because of the extra costs if causing an accident, although lost time and work may motivate an employed driver to go faster. This result may also help explain why teenage drivers tend to speed more and driver more reckless.

[10]Traffic policy makers have recognized this manifestation of driver frustration by initiating so-called "traffic calming" initiatives, including speed bumps, speed dips, and, ironically, manipulating the phases of traffic signals.

[11]With perfect information on the part of both drivers and engineers, homogeneous driving and automobile characteristics, and zero response times by drivers, the DZ proposed by GHM is of zero width, eliminating all possibility of unintentional red light violations.

[12]Figure 1 is borrowed from Bennink (1996).

[13]Despite traffic engineer's good intentions, one would be hard pressed to find many drivers who are aware of these markings.

[14]A driver in the dilemma zone may successfully cross the intersection if she can accelerate

sufficiently to cross before her light turns red; we do not model that situation here. Secondly, a driver may run a red light but not cause a collision if cross traffic does not enter the intersection immediately upon their light turning green.

[15]This reduction is not to be confused with using traffic signals to reduce average velocity attained, as mentioned in the introduction.

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Table 1
Dilemma Zone Based Upon Only Different Approach Speeds

Velocity	X_C	X_0	Dilemma Zone (DZ)	Time to Cross DZ
35 mph = 57.75 ft/s	224.50 ft	181.79 ft	42.71 ft	0.73 sec
40 mph = 66.00 ft/s	283.80 ft	217.76 ft	66.04 ft	1.00 sec
42 mph = 69.30 ft/s	309.42 ft	232.14 ft	77.27 ft	1.11 sec
45 mph = 75.25 ft/s	349.90 ft	253.14 ft	96.17 ft	1.29 sec
50 mph = 82.50 ft/s	422.81 ft	289.70 ft	133.11 ft	1.61 sec

Engineer's Driver: $V_0 = 40mph = 66ft/s$, $a_2 = 10ft/s^2$, $\Lambda_0 = 20ft$, $\omega = 50ft$
Amber phase set at 4.36 sec according to $T_A = \frac{V_0}{2a_2} + \frac{\omega+\Lambda_0}{V_0}$
Hypothetical driver matches engineer's driver except for approach velocity
and reaction time $s_{i2} = 1\ sec$.

Table 2
Dilemma Zone Based Upon Different Approach Speeds and Reaction Time

Velocity	X_C	X_0	Dilemma Zone (DZ)	Time to Cross DZ
35 mph = 57.75 ft/s	195.62 ft	181.79 ft	13.83 ft	0.23 sec
40 mph = 66.00 ft/s	250.8 ft	217.76 ft	33.04 ft	0.50 sec
42 mph = 69.30 ft/s	274.77 ft	232.14 ft	42.63 ft	0.61 sec
45 mph = 75.25 ft/s	320.75 ft	253.14 ft	67.61 ft	0.89 sec
50 mph = 82.50 ft/s	381.56 ft	289.70 ft	91.86 ft	1.11 sec

Engineer's Driver: $V_0 = 40mph = 66ft/s$, $a_2 = 10ft/s^2$, $\Lambda_0 = 20ft$, $\omega = 50ft$
Amber phase set at 4.36 sec according to $T_A = \frac{V_0}{2a_2} + \frac{\omega+\Lambda_0}{V_0}$
Hypothetical driver matches engineer's driver except for approach velocity
and reaction time $s_{i2} = 0.5\ sec$.

Table 3
Dilemma Zone Based Upon Different Approach Speeds and Deceleration Constant

Velocity	X_C	X_0	Dilemma Zone (DZ)	Time to Cross DZ
35 mph = 57.75 ft/s	168.91 ft	181.79 ft	NA	NA
40 mph = 66.00 ft/s	211.20 ft	217.76 ft	NA	NA
42 mph = 69.30 ft/s	229.38 ft	232.14 ft	NA	NA
45 mph = 75.25 ft/s	258.01 ft	253.14 ft	4.87 ft	0.06 sec
50 mph = 82.50 ft/s	309.37 ft	289.70 ft	19.67 ft	0.23 sec

Engineer's Driver: $V_0 = 40mph = 66ft/s$, $a_2 = 10ft/s^2$, $\Lambda_0 = 20ft$, $\omega = 50ft$
Amber phase set at 4.36 sec according to $T_A = \frac{V_0}{2a_2} + \frac{\omega+\Lambda_0}{V_0}$
Hypothetical driver matches engineer's driver except for approach velocity, $a = 15ft/s^2$, and reaction time $s_{i2} = 1\ sec$.

Table 4
Dilemma Zone Based Upon Different Approach Speeds and Vehicle Length

Velocity	X_C	X_0	Dilemma Zone (DZ)	Time to Cross DZ
35 mph = 57.75 ft/s	224.50 ft	178.79 ft	45.71 ft	0.79 sec
40 mph = 66.00 ft/s	283.80 ft	214.76 ft	69.04 ft	1.04 sec
42 mph = 69.30 ft/s	309.42 ft	229.14 ft	80.27 ft	1.15 sec
45 mph = 75.25 ft/s	349.90 ft	249.14 ft	99.17 ft	1.31 sec
50 mph = 82.50 ft/s	422.81 ft	286.70 ft	136.11 ft	1.65 sec

Engineer's Driver: $V_0 = 40mph = 66ft/s$, $a_2 = 10ft/s^2$, $\Lambda_0 = 20ft$, $\omega = 50ft$
Amber phase set at 4.36 sec according to $T_A = \frac{V_0}{2a_2} + \frac{\omega+\Lambda_0}{V_0}$
Hypothetical driver matches engineer's driver except for approach velocity, $\Lambda = 23ft$, and reaction time $s_{i2} = 1\ sec$.

Table 5
Dilemma Zone Based Upon Different Approach Speeds and Reduced Amber Phase

Velocity	X_C	X_0	Dilemma Zone (DZ)	Time to Cross DZ
35 mph = 57.75 ft/s	224.50 ft	161.00 ft	63.50 ft	1.09 sec
40 mph = 66.00 ft/s	283.80 ft	194 ft	89.80 ft	1.36 sec
42 mph = 69.30 ft/s	309.42 ft	207.2 ft	102.22 ft	1.47 sec
45 mph = 75.25 ft/s	349.90 ft	227.00 ft	122.90 ft	1.65 sec
50 mph = 82.50 ft/s	422.81 ft	260.00 ft	162.81 ft	1.97 sec

Engineer's Driver: $V_0 = 40mph = 66ft/s$, $a_2 = 10ft/s^2$, $\Lambda_0 = 20ft$, $\omega = 50ft$
Amber phase set at 4.00 sec rather than 4.36 sec according to $T_A = \frac{V_0}{2a_2} + \frac{\omega+\Lambda_0}{V_0}$
Hypothetical driver matches engineer's driver except for approach velocity,
and reaction time $s_{i2} = 1$ *sec*.

Figure 1: Light Phases Under Ideal Conditions
Dilemma Zone is of Zero Width: No inadvertent Red Light Violations

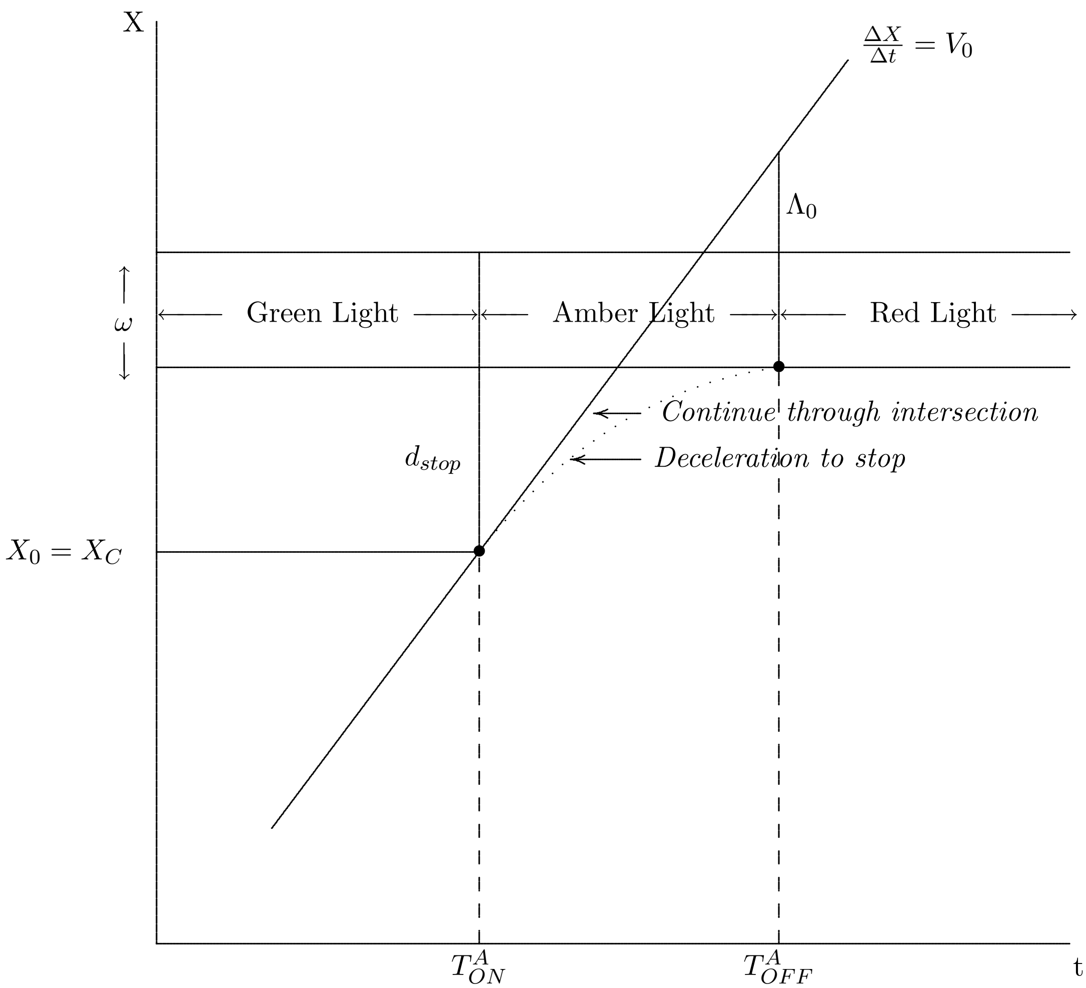

Figure 2: The Dilemma Zone
Inadvertent Red Light Violations Possible in the Dilemma Zone